Essential Maths Skills

for AS/A-level

Design and Technology Answers

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1.2 Working with formulae and equations

Guided questions (p. 19)

1 Step 1: 1 cm is 1.0×10^{-2} m

400 cm = 400×10^{-2} m = 4.00 m

Note: 400 could be accurate to 1, 2 or 3 significant figures — 3 significant figures have been kept here after unit conversion.

Step 2: 400 cm × 1.2 m = 4.00 m × 1.2 m = 4.80 m^2

However, the least accurate value, 1.2 m, is significant to 2 figures only, so the answer should be provided to this accuracy.

4.80 m^2 = 4.8 m^2 (2 s.f.)

2 Step 1: 0.020 A = 2.0×10^{-2} A

Step 2: $V = IR = 2.0 \times 10^{-2}\text{ A} \times 330\,\Omega = 660 \times 10^{-2}\text{ V}$

Step 3: $660 \times 10^{-2}\text{ V} = 6.60 \times 10^{2} \times 10^{-2}\text{ V}$

When multiplying powers of 10, add the powers.

$6.60 \times 10^{2} \times 10^{-2}\text{ V} = 6.60\text{ V}$

The most accurate value given is 0.020 A, which has two significant figures, so change the answer to this level of accuracy.

Answer: 6.60 V = 6.6 V (2 s.f.)

3 Step 1: 12 mm = 12×10^{-3} m = 1.2×10^{-2} m

Step 2: volume of a cylinder = $\pi r^2 h = \pi(1.2 \times 10^{-2})^2 \times 0.2$

$\pi(1.2 \times 10^{-2})^2 \times 0.2 = \pi(1.44 \times 10^{-4}) \times 0.2 = 0.90478 \times 10^{-4}$ (5 d.p.)

Step 3: The values provided at the start of the calculation are accurate to 2 significant figures, so this answer should be given to the same level of significance.

Answer: 0.90×10^{-4} (2 s.f.)

Practice questions (p. 19)

4 The footprint of the laptop is the area of the rectangular base, i.e. length × width.

area of a rectangle = length × width

Substitute the values:

area = 360 mm × 250 mm = 3.6×10^{-1} m × 2.5×10^{-1} m = $9 \times 10^{-2}\text{ m}^2$

Answer: Area = $9 \times 10^{-2}\text{ m}^2$

5 Use the density formula:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{v}$$

Substitute the values for mass and volume.

$$\rho = \frac{m}{v} = \frac{1}{3.5 \times 10^{-2}} = 2857 = 2.9 \times 10^3\,\text{kg/m}^3\,(2\text{ s.f.})$$

Answer: The density of the base material needs to be a minimum of $2.9 \times 10^3\,\text{kg/m}^3\,(2\text{ s.f.})$.

6 The power formula will be required for this calculation:

P (watts) = V (volts) × I (amperes)

First convert the current into standard form. When you have calculated the power, convert it into standard form, then use an appropriate prefix to best present the answer.

$$20\,\text{mA} = 20 \times 10^{-3}\,\text{A} = 2.0 \times 10^{-2}\,\text{A}$$

$$P = V \times I = 2.1\text{V} \times 2.0 \times 10^{-2}\,\text{A} = 4.2 \times 10^{-2}\,\text{W} = 42\text{ milliwatts} = 42\,\text{mW}$$

Answer: 42 mW

1.3 Equations of motion

(OCR Design Engineering only, *online*)

Guided question (p. 7)

1 Step 1: $u = 3.5\,\text{m}\,\text{s}^{-1}$

$v = 14.2\,\text{m}\,\text{s}^{-1}$

$s = 75\,\text{m}$

Step 2: $v^2 = u^2 + 2as$

Step 3: Rearrange the equation and substitute the values to find a.

$$v^2 - u^2 = 2as$$

$$a = \frac{v^2 - u^2}{2s}$$

$$a = \frac{14.2^2 - 3.5^2}{2 \times 75} = \frac{201.64 - 12.25}{150}$$

$a = 1.26\,\text{m}\,\text{s}^{-2}$ (to 2 d.p.)

1.4 Scientific and engineering formulae

(OCR Design Engineering only, *online*)

Guided questions (p. 12)

1 Step 1: $P = \frac{E}{t}$

Rearrange the formula in terms of energy (E):

$$E = P \times t$$

$E = 18 \times (10 \times 60)$ (there are 60 seconds in each minute)

$$E = 10\,800\,\text{J}$$

Step 2: $E = mc\Delta T$

Rearrange the formula in terms of temperature rise (ΔT):

$$\Delta T = \frac{E}{mc}$$

Use the energy value calculated in step 1:

$$\Delta T = \frac{10800}{0.2 \times 910} \quad \text{(200 g is equivalent to 0.2 kg)}$$

$\Delta T = 59.3°C$ (to 1 d.p.)

2 Step 1: $W = mg$

$W = 5 \times 9.81$

$W = 49.05\,N$

Step 2: $A = \frac{\pi d^2}{4}$

$$A = \pi \times \frac{(1 \times 10^{-3})^2}{4} \quad (1\,mm = 1 \times 10^{-3}\,m)$$

$A = 7.85 \times 10^{-7}\,m^2$ (to 2 d.p.)

Step 3: $\text{stress} = \frac{\text{force}}{\text{cross-sectional area}}$

$$\text{stress} = \frac{49.05}{7.85 \times 10^{-7}} \quad \text{(the tensile force in the wire is equal to the applied weight)}$$

$\text{stress} = 62.5 \times 10^6\,Pa$

Step 4: Young's modulus, $E = \frac{\text{stress}}{\text{strain}}$

$$\text{strain} = \frac{\text{stress}}{E} \quad \text{(use the stress value calculated in step 3)}$$

$$\text{strain} = \frac{62.5 \times 10^6}{200 \times 10^9} \quad (200\,GPa = 200 \times 10^9\,Pa)$$

$\text{strain} = 3.125 \times 10^{-4}$ (strain does not have any units)

Step 5: $\text{strain} = \frac{\text{extension}}{\text{orginal length of wire}}$

extension = strain × original length

extension = $3.125 \times 10^{-4} \times 1.8$

extension = $5.63 \times 10^{-4}\,m$ (to 2 d.p) = 0.563 mm

Practice questions (p. 12)

3 Step 1: Write down the parameters that are given in the question:

$u = 0$

$a = 120\,m\,s^{-2}$

$t = 0.6\,s$

Step 2: We need to find v. Select the equation of motion that contains these parameters:

$v = u + at$

Step 3: Substitute in the values to calculate v:

$v = 0 + (120 \times 0.6)$

$v = 72\,\text{m}\,\text{s}^{-1}$

4 Step 1: Write down the parameters that are given in the question:

$t = 2\,\text{s}$

$a = g = 9.81\,\text{m}\,\text{s}^{-2}$

$u = 0$

The question is asking us to calculate the distance the package will fall in 2 s under freefall. Therefore, we need to find s.

As this is a freefall problem involving only downwards motion, use the convention that the downwards direction is positive, therefore g is positive.

Step 2: Select the equation of motion that contains these parameters:

$s = ut + \frac{1}{2}at^2$

Step 3: Substitute in the values to calculate s:

$s = 0 + (\frac{1}{2} \times 9.81 \times 2^2)$

$s = 19.62\,\text{m}$

5 **a** Step 1: Write down the parameters that are given in the question:

$m = 200\,\text{kg}$

$v = 13\,\text{m}\,\text{s}^{-1}$

Step 2: Use the kinetic energy formula:

$E = \frac{1}{2}mv^2$

Step 3: Substitute in the values:

$E = \frac{1}{2} \times 200 \times 13^2$

$E = 16\,900\,\text{J}$

b Step 1: To bring the motorbike to a stop, all the kinetic energy needs to be transferred to heat energy in the brakes. Therefore, the brakes need to 'do work' by applying a retarding force F to the motorbike over a distance d.

work done, $W = F \times d$

Step 2: Rearrange in terms of F:

$F = \dfrac{W}{d}$

$d = \dfrac{16\,900}{20}$ (the work done equals the kinetic energy calculated above)

$d = 845\,\text{N}$

6 Step 1: Calculate the weight of the 100 g mass, as this will be the force that is applied to the spring:

$W = mg$

$W = 0.1 \times 9.81$ (100 g is equal to 0.1 kg)

$W = 0.981$ N

Step 2: Use the spring equation (see Table 1.7, page 18) to calculate the extension:

spring force = spring constant × extension

$F = ke$

Step 3: Rearrange in terms of extension, e:

$$e = \frac{F}{k}$$

Step 4: Substitute the values:

$$e = \frac{0.981}{300}$$

$e = 3.27 \times 10^{-3}$ m = 3.27 mm

2 Ratios and percentages

2.1 Ratios and scaling of lengths, area and volume

Guided questions (p. 28)

1 The area scaling ratio is:

original area : new area

$108\,mm^2 : 200\,mm^2$

Dividing both sides by 4 gives:

27 : 50

Calculate the scaling ratio from the area scaling ratio:

$\sqrt{27} : \sqrt{50}$

Neither of these values has a square root that is a whole integer, so for now leave them written as roots. The ratio of:

original side : new side is the same as $\sqrt{27} : \sqrt{50}$

Therefore, the new side dimensions will be:

$$\frac{\text{new side}}{\text{original side}} = \frac{\sqrt{50}}{\sqrt{27}}$$

Rearranging gives:

$$\text{new side} = \frac{\sqrt{50}}{\sqrt{27}} \times \text{original side}$$

Now use this formula to calculate the lengths of the sides.

$$\text{new width} = \frac{\sqrt{50}}{\sqrt{27}} \times 12\,\text{mm} = 16.33\,\text{mm}\,(2\text{ d.p.})$$

$$\text{new height} = \frac{\sqrt{50}}{\sqrt{27}} \times 9\,\text{mm} = 12.25\,\text{mm}\,(2\text{ d.p.})$$

Check: Multiplying these two values should give the new area of $200\,mm^2$.

$16.33 \times 12.25 = 200$

which proves the values are correct.

Answer: The new width is 16 mm (2 s.f.).

The new height is 12 mm (2 s.f.).

2 Step 1: The volume scaling ratio is original volume : new volume

$96 : 150$

Dividing both sides by 6 gives: $16 : 25$

Scaling ratio for the height is: $\sqrt[3]{16} : \sqrt[3]{25}$

Step 2: Calculate the new height.

$$\frac{\text{new height}}{\text{original height}} = \frac{\sqrt[3]{25}}{\sqrt[3]{16}}$$

$$\text{new height} = \frac{\sqrt[3]{25}}{\sqrt[3]{16}} \times \text{original height} = \frac{\sqrt[3]{25}}{\sqrt[3]{16}} \times 4\,\text{mm} = 4.64\,\text{mm (2 d.p.)}$$

Answer: The new height is 4.64 mm (2 d.p.).

3 Step 1: The cost scaling ratio is:

£0.15 : £2.10

which is the same as 1 : 14 because 15 pence goes into £2.10 14 times.

For any scaling ratio $1 : n$, the scaling ratio of the area is $1 : n^2$.

Step 2: As the cost scaling ratio is proportional to the area scaling ratio, the dimensions scaling ratio is:

$1 : \sqrt{14}$

Therefore, the new width is:

$$\text{new width} = \text{original width} \times \sqrt{14} = 12 \times \sqrt{14} = 45\,\text{mm}\ (2\text{ s.f.})$$

$$\text{new height} = \text{original height} \times \sqrt{14} = 9 \times \sqrt{14} = 34\,\text{mm}\ (2\text{ s.f.})$$

Answer: The width of the larger rectangle of cotton fabric is 45 mm (2 s.f.).

The height of the larger rectangle of cotton fabric is 34 mm (2 s.f.).

Practice questions (p. 28)

4 The scale ratio 2 : 3 is the same as the ratio original height : new height. Therefore the two ratios are equal and:

$$\frac{\text{new height}}{\text{original height}} = \frac{3}{2}$$

Rearranging this gives:

$$\text{new height} = \frac{3}{2} \times \text{original height} = \frac{3}{2} \times 600 = 900\,\text{mm}$$

Answer: The new height is 900 mm.

5 The scale ratio of the heights is 400 : 600 or 2 : 3.

For any scaling ratio $x : y$, the scaling ratio of the volume is $x^3 : y^3$. Therefore the scaling ratio of the volume is:

$2^3 : 3^3$

$8 : 27$

Answer: The volume scaling ratio is 8 : 27.

6 Mass is proportional to volume. Therefore the scaling ratios are the same.

$0.4 : 1.4$

Dividing by 2 gives: $0.2 : 0.7$

For any scaling ratio $x : y$, the scaling ratio of the volume is $x^3 : y^3$.

Therefore: $\frac{\text{new height}}{\text{original height}} = \frac{\sqrt[3]{0.7}}{\sqrt[3]{0.2}}$

Rearranging gives:

$$\text{new height} = \frac{\sqrt[3]{0.7}}{\sqrt[3]{0.2}} \times \text{original height} = \frac{\sqrt[3]{0.7}}{\sqrt[3]{0.2}} \times 4\,\text{mm} = 6.07\,\text{mm (2 d.p.)}$$

Answer: The height of the cuboid with the larger mass is 6 mm (1 s.f.).

2.2 Ratios and mechanisms

(OCR Design Engineering only, *online*)

Guided question (p. 16)

1 a $\text{gear ratio} = \frac{\text{number of teeth on driven gear}}{\text{number of teeth on driver gear}}$

Step 1: $\text{gear ratio 1} = \frac{12}{36}$

$\text{gear ratio 1} = \frac{1}{3}$

Step 2: $\text{gear ratio 2} = \frac{10}{40}$

$\text{gear ratio 2} = \frac{1}{4}$

Step 3: overall gear ratio = gear ratio 1 × gear ratio 2

$$\text{overall gear ratio} = \frac{1}{3} \times \frac{1}{4}$$

$$\text{overall gear ratio} = \frac{1}{12}$$

b Step 1: $\text{gear ratio} = \frac{\text{input speed}}{\text{output speed}}$

Step 2: Rearranging:

$$\text{output speed} = \frac{\text{input speed}}{\text{gear ratio}}$$

$$\text{output speed} = \frac{100}{\left(\frac{1}{12}\right)} = 100 \times 12 = 1200\,\text{rpm}$$

Practice questions (p. 16)

2 **a** The pliers are a pair of levers (we only have to consider one side of the pliers). For a lever, mechanical advantage (MA) is equal to the ratio of input and output arm lengths:

$$\text{MA} = \frac{\text{input arm length}}{\text{output arm length}}$$

$$\text{MA} = \frac{120}{40} = 3$$

b For any mechanical system:

$$\text{MA} = \frac{\text{load}}{\text{effort}}$$

Rearrange to find the load:

$$\text{load} = \text{MA} \times \text{effort}$$

$$\text{load} = 3 \times 75$$

$$\text{load} = 225\,\text{N}$$

3 Step 1: Calculate the mechanical advantage required:

$$\text{MA} = \frac{\text{load}}{\text{effort}}$$

$$\text{MA} = \frac{1000}{50} = 20$$

The mechanical advantage is also equal to the ratio of the lever arm lengths:

$$\text{MA} = \frac{\text{input arm length}}{\text{output arm length}}$$

Step 2: Rearrange to find the input arm length:

$$\text{input arm length} = \text{MA} \times \text{output arm length}$$

$$\text{input arm length} = 20 \times 30 = 600\,\text{mm}$$

4 **a** $$\text{gear ratio} = \frac{\text{input speed}}{\text{output speed}}$$

$$\text{gear ratio} = \frac{3000}{600}$$

$$\text{gear ratio} = 5$$

b $$\text{gear ratio} = \frac{\text{number of teeth on driven gear}}{\text{number of teeth on driver gear}}$$

The gear on the motor is the driver gear (the input gear). Rearrange the gear ratio equation to find number of teeth on driven gear:

$$\text{teeth on driven gear} = \text{gear ratio} \times \text{teeth on driver gear}$$

$$\text{teeth on driven gear} = 5 \times 9 = 45 \text{ teeth}$$

5 Step 1: Calculate the gear ratio required:

$$\text{gear ratio} = \frac{\text{input speed}}{\text{output speed}}$$

$$\text{gear ratio} = \frac{90}{270} = \frac{1}{3}$$

The gear connected to the wind turbine is the input gear. We need to calculate the number of teeth on the output gear:

$$\text{gear ratio} = \frac{\text{teeth on output gear}}{\text{teeth on input gear}}$$

Step 2: Rearrange to find teeth on output gear:

$$\text{teeth on output gear} = \text{gear ratio} \times \text{teeth on input gear}$$

$$\text{teeth on output gear} = \frac{1}{3} \times 48 = 16 \text{ teeth}$$

6 For a compound gear train:

$$\text{overall gear ratio} = \text{gear ratio of stage 1} \times \text{gear ratio of stage 2}$$

If each stage is identical then they both have the same gear ratio, so the above equation can be written:

$$\text{overall gear ratio} = (\text{gear ratio of each stage})^2$$

Therefore, the gear ratio of each stage will be the square root of the overall gear ratio:

$$\text{gear ratio of each stage} = \sqrt{\text{overall gear ratio}}$$

$$\text{gear ratio of each stage} = \sqrt{20.25} = 4.5$$

2.3 Working with percentages

Guided questions (p. 32)

1 Option 1: Use the formula for percentage decrease:

$$\text{new value} = \frac{100 - \text{percentage decrease}}{100} \times \text{original value}$$

Rearrange the formula into terms of percentage decrease. Then substitute the values.

Step 1: Divide both sides by the original value.

$$\frac{\text{new value}}{\text{original value}} = \frac{100 - \text{percentage decrease}}{100} \times \frac{\cancel{\text{original value}}}{\cancel{\text{original value}}}$$

Step 2: Multiply both sides by 100.

$$\frac{\text{new value}}{\text{original value}} \times 100 = \frac{100 - \text{percentage decrease}}{\cancel{100}} \times \cancel{100}$$

Step 3: Add percentage decrease to both sides.

$$\text{percentage decrease} + \frac{\text{new value}}{\text{original value}} \times 100$$

$$= 100 - \cancel{\text{percentage decrease} + \text{percentage decrease}}$$

Step 4: Subtract the fraction from both sides to leave just percentage decrease on the right-hand side.

$$\text{percentage decrease} = 100 - \frac{\text{new value}}{\text{original value}} \times 100$$

Substituting values gives:

$$\text{percentage decrease} = 100 - \frac{200}{210} \times 100 = 4.76\% \text{ (3 s.f.)}$$

Answer: The linkage has decreased in length by 4.76% (3 s.f.).

Option 2: Step 1: Divide the new value by the original value.

$$\frac{\text{new value}}{\text{original value}} = \frac{200}{210} = 0.95238\ (5\text{ s.f.})$$

Step 2: Subtract the decimal from 1.

$1 - 095238 = 0.0476.$

Step 3: Multiply by 100.

$0.0476 \times 100 = 4.76\%$

Answer: The linkage has decreased in length by 4.76% (3 s.f.).

2 Step 1:

area of a rectangle = width × height = $1.2 \times 1.2 = 1.44\,\text{m}^2$

Step 2:

$$\text{area of circle} = \pi r^2 = \pi \times 0.6^2 = 1.13\,\text{m}^2\ (3\text{ s.f.})$$

Step 3:

$$\%\text{ waste} = \frac{\text{value including waste} - \text{value excluding waste}}{\text{value including waste}} \times 100$$

$$\%\text{ waste} = \frac{1.44 - 1.13}{1.44} \times 100 = 21.5\%\ (3\text{ s.f.})$$

Answer: 21.5% of the plywood is waste.

3 Step 1:

$$\text{percentage faulty} = \frac{\text{number of faulty circuit boards}}{\text{total number of circuit boards}} \times 100$$

Step 2: Substituting values gives:

$$0.6 = \frac{x}{1200} \times 100$$

$$x = \frac{0.6 \times 1200}{100} = 7.2$$

Answer: 7 of the circuit boards were faulty.

Practice questions (p. 32)

4 To calculate the percentage waste, divide the waste by the total and multiply by 100.

$$\%\text{ waste} = \frac{\text{value including waste} - \text{value excluding waste}}{\text{value including waste}} \times 100$$

$$\%\text{ waste} = \frac{\text{waste}}{\text{value including waste}} \times 100$$

First, calculate the waste:

waste = $1.32 - 1.21 = 0.11\,\text{m}^2$

$$\text{percentage waste} = \frac{0.11}{1.32} \times 100 = 8.33 = 8.33\%\ (2\text{ d.p.})$$

Answer: Percentage waste = 8.33% (2 d.p.).

5 Step 1: Calculate the cost to manufacture each desk lamp.

If the profit is 35% for a £12.00 per unit sale, then £12.00 represents 135%.

So the cost of producing one lamp is:

$$\frac{100}{135} \times 12 = £8.89$$

Step 2: Calculate the selling price for the larger quantity order.

£8.89 + £3.00 = £11.89

Answer: For orders over 100 units the individual desk lamp selling price is £11.89.

6 Step 1: The percentage error formula can be used for this calculation. The measured value is the actual concrete required and because more concrete is required, the value must be larger than the original expected value.

$$\% \text{ error} = \frac{\text{measured value} - \text{expected value}}{\text{expected value}} \times 100$$

$$\% \text{ error} = \frac{\text{actual volume of concrete} - \text{expected volume of concrete}}{\text{expected volume of concrete}} \times 100$$

Step 2: Rearranging to make the subject of the formula the actual volume of concrete gives:

$$\text{actual volume of concrete} = \frac{\% \text{ error} \times \text{expected volume of concrete}}{100} + \text{expected volume of concrete}$$

Step 3: Calculate the expected volume of concrete.

Total expected volume of concrete = 1.5 bags × $9.0 \times 10^{-3}\,\text{m}^3 = 13.5 \times 10^{-3}\,\text{m}^3$
$= 1.35 \times 10^{-2}\,\text{m}^3$

Step 4: Substitute values and calculate the answer.

$$\text{actual volume of concrete} = \frac{12 \times 1.35 \times 10^{-2}}{100} + 1.35 \times 10^{-2} = 1.512 \times 10^{-2} = 1.5 \times 10^{-2}\,\text{m}^3 (2 \text{ s.f.})$$

Answer: The volume of concrete required is $1.5 \times 10^{-2}\,\text{m}^3$ (2 s.f.).

3 Calculating surface areas and volumes

3.1 Properties and areas of two-dimensional shapes

Guided questions (p. 41)

1 From Section 3.1 use:

$$\text{area of parallelogram} = \text{width, } w \times \text{perpendicular height, } h$$
$$= 1.2\,\text{m} \times 0.6\,\text{m} = 0.72\,\text{m}^2$$

Answer: The surface area of the desktop will be $0.72\,\text{m}^2$.

2 Step 1: area of wood = $440 \times 200 = 88\,000\,\text{mm}^2$

Step 2: The waste area $= ((40 \times 40) - \pi \times 20^2) + \pi \times 20^2 + (140 \times 40) = 7200.00\,\text{mm}^2$ to 2 d.p.

$$\text{percentage waste} = \frac{\text{waste area}}{\text{original area of wood}} \times 100 = \frac{7200}{88\,000} \times 100$$
$$= 8.2\% \text{ to the nearest whole \%.}$$

Answer: The waste material is 8.2% of the original wood.

Practice questions (p. 42)

3 Step 1: Calculate the thickness of the laminate in order to calculate the increase to each of the radii. The large radius is half of the diameter 776 mm = 388 mm.

Laminate thickness = 8 layers × 1.5 mm = 12 mm

New large radius = 388 mm + 12 mm = 400 mm

New small radius = 188 mm + 12 mm = 200 mm

Step 2: Calculate the length of the curve with the large radius. The curve is one quarter of a circle.

$$\text{length of arc} = \frac{\text{perimeter of circle}}{4} = \frac{2\pi \times 400}{4} = 628\text{mm} \ (3 \text{ s.f.})$$

Step 3: Calculate the length of the curve with the small radius. This curve is three-quarters of a circle.

$$\text{length of arc} = \frac{3}{4} \times \text{perimeter of circle} = \frac{3}{4} \times 2\pi \times 200 = 942\,\text{mm} \ (3 \text{ s.f.})$$

Step 4: Add the lengths of the curves and the straight to find the answer.

length of outer layer = 628 + 600 + 942 = 2170 mm (3 s.f.)

Answer: The length of the outer layer is 2170 mm (3 s.f.).

4 **a** Step 1: Calculate the time taken for a single pass of 1200 mm.

A single 1200 mm pass of the cutter = $\frac{6 \text{ min}}{3} = 2 \text{ min}$

Step 1: Convert values into base units of m and s.

1200 mm = 1.2 m

2 min = 120 seconds

Step 3: Calculate the speed.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{1.2\text{ m}}{120 \text{ seconds}} = 0.01 \text{ m s}^{-1}$$

Answer: The CNC router cutting speed is 0.01 m s^{-1}.

b Step 1: Calculate the distance covered by the router, i.e. the perimeter of the leg. Remember to multiply this by 3 to include the three passes.

From the workings, the perimeter is 5800 mm or 5.8 m. Therefore, the total distance covered in all 3 passes is $3 \times 5.8\text{ m} = 17.4\text{ m}$.

Step 1: Calculate the time taken to travel this distance.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Rearranging gives:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Substituting values gives:

$$\text{time} = \frac{17.4\text{ m}}{0.01 \text{ m s}^{-1}} = 1740 \text{ seconds}$$

Convert into minutes by dividing by 60 seconds:

$$1740 \text{ seconds} = \frac{1740}{60}\text{ minutes} = 29 \text{ minutes}$$

Answer: It takes 29 minutes to cut out the table leg from the sheet of plywood.

3.2 Surface area of three-dimensional objects

Guided question (p. 47)

1 Step 1: Calculate the curved surface area.

curved surface area of parallel truncated cone = $\pi S (r + R)$

Substitute values:

curved surface area of parallel truncated cone = $\pi \times 50\ (20 + 30) = 7854 \text{ mm}^2$ (nearest mm^2)

Step 1: Calculate the area of the base.

$$\text{area of circle} = \frac{\text{circumference} \times \text{radius}}{2} = \frac{2\pi r \times r}{2} = \pi r^2$$

Substitute values:

$$\text{area of circle} = \pi \times 20^2 = 1257\,\text{mm (nearest mm}^2\text{)}$$

Step 3: Add the values to find the total surface area painted.

Total surface area painted = 7854 + 1257 = 9111 mm^2 = 9.1×10^3 mm^2 (2 s.f.)

Answer: The surface area coated with blue enamel paint is 9.1×10^3 mm^2 (2 s.f.).

Practice questions (p. 47)

2 To calculate the percentage of waste, do the following:

$$\text{percentage waste} = \frac{\text{area of waste}}{\text{total area}} \times 100$$

Step 1: Calculate the area of waste.

Total waste = sum of the holes

$= (12\pi \times 3^2) + (\pi \times 7.5^2) + (2\pi \times 1.5^2) + (\pi \times 5^2)$
$= 609\,\text{mm}^2$ (nearest mm^2)

Step 2: Calculate the total area of mild steel sheet.

Total area = surface area of the outer curved surface of the cylinder + curved surface area of hemisphere

surface area of the curved surface of a cylinder = circumference of base × height

$= 2\pi rh = 2\pi \times 40 \times 80 = 20\,106\,\text{mm}^2$ (nearest whole mm^2)

Curved surface area of a hemisphere = $2\pi r^2 = 2 \times \pi \times 40^2 = 10\,053\,\text{mm}^2$

Total area = 20 106 + 10 053 = 30 159 mm^2

Step 3: Calculate the percentage waste by substituting the values.

$$\text{percentage waste} = \frac{609}{30159} \times 100 = 2.0\%\ (2\text{ s.f.})$$

Answer: The percentage waste is 2.0% (2 s.f.).

3 This looks like a complicated question, but when broken down it becomes easier. You can use Figure 3.24 again to help and some of the work has already been done in your Section 3.1 answer.

First, use the results from Section 3.1 to calculate the surface area of the top and bottom faces.

Top or bottom face area = wood area – waste area = 88 000 mm^2 – 7200 mm^2
= 80 800 mm^2

The four outside corner curved faces make a complete cylinder, as do the semi-circular faces in the middle of the top of the stool. These are the same radius of 20 mm and the same thickness of 20 mm.

Cylinder surface area = $2\pi rh = 2\pi \times 20 \times 20 = 2513.3\,\text{mm}^2$ to 1 d.p.

Next, calculate the surface areas of the rectangular faces of the stool top. A sketch will help you – see Figure A.1.

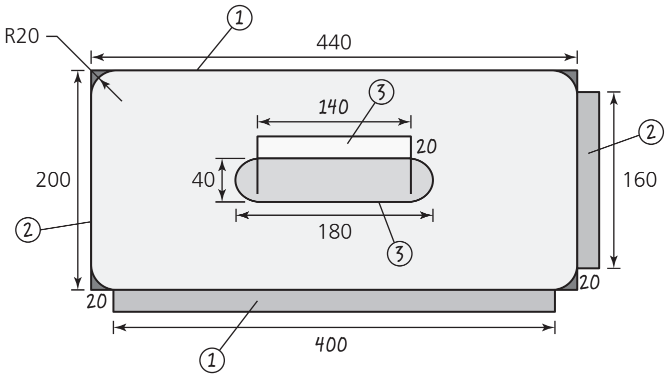


Figure A.1

Colour can be used to help you identify the different faces. A development, or part of a development, as shown, can help you identify the shape of the faces. From the drawing, the surface areas of the rectangular faces are:

Face 1: area = $400 \times 20 = 8000\,mm^2$

Face 2: area = $160 \times 20 = 3200\,mm^2$

Face 3: area = $140 \times 20 = 2800\,mm^2$

Finally, add up all of the surface areas.

Total surface area = $(2 \times 80\,800) + (2 \times 2513.3) + (2 \times 8000) + (2 \times 3200) + (2 \times 2800) = 194\,626.6\,mm^2 = 0.194627\,m^2 = 0.195\,m^2$ to 3 d.p.

Answer: One coat of paint would need to cover $0.195\,m^2$ to 3 d.p.

3.3 Volume of three-dimensional objects

Guided question (p. 53)

1 Step 1:

i Volume of 'external' cuboid = $135 \times 70 \times 12 = 113\,400\,mm^3$

ii Volume of waste at corners = volume of cuboid – volume of cylinder = $(20 \times 20 \times 12) - (\pi \times 10^2 \times 12) = 4800 - 3770 = 1030\,mm^3$ (nearest mm^3)

iii Volume of 'external' cuboid = $113\,400 - 1030 = 112\,370\,mm^3$ (nearest mm^3)

Step 2:

i Volume of 'internal' cuboid = $131 \times 66 \times 10 = 86\,460\,mm^3$

ii Volume of waste at corners = volume of cuboid – volume of cylinder = $(16 \times 16 \times 10) - (\pi \times 8^2 \times 10) = 2560 - 2011 = 549\,mm^3$ (nearest mm^3)

iii Volume of 'internal' cuboid = $86\,460 - 549 = 85\,911\,mm^3$ (nearest mm^3)

Step 3:

i Volume of acrylic required = $112\,370 - 85\,911 = 26\,459\,mm^3$ (nearest mm^3)

Answer: Volume of acrylic required = $26\,459\,mm^3$ (nearest mm^3).

Practice question (p. 54)

2 Step 1: Convert all dimensions to the same base units of metres.

10 cm = 0.1 m

Step 2: Identify that for the minimum volume to be achieved, there shouldn't be any rubber chippings beneath the roundabout. So, the rubber chippings are covering only a 1.2 m strip around the roundabout.

Step 3: Calculate the surface area of rubber chippings required.

The surface area of the surrounding strip = the large circle – the inner circle.

To calculate the areas of the circles, the respective radii are required.

$$\text{radius of inner circle} = \frac{3.2\,\text{m}}{2} = 1.6\ \text{m}$$

radius of outer circle = 1.6 m + 1.2 m = 2.8 m

$$A_{\text{large}} = \pi \times 2.8^2 = 24.63\text{m}^3$$

$$A_{\text{small}} = \pi \times 1.6^2 = 8\ .04\,\text{m}^3$$

Therefore $A_{\text{strip}} = 24.63 - 8.04 = 16.59\text{m}^3 = 16.6\text{m}^3$ (to 3 s.f.)

Step 4: Calculate the volume of the strip.

volume of a cylinder = base area × height

So, volume of strip = area of strip × depth of rubber chippings = $A_{\text{strip}} \times 0.1 = 16.6 \times 0.1$ = 1.66 m^3.

Answer: Volume of rubber chippings required = 1.66 m^3 to 3 s.f.

3.4 Density and mass of three-dimensional objects

Guided questions (p. 57)

1 Step 1: volume of ABS = 5600 $\text{mm}^3 = 5.6 \times 10^3 \times 10^{-9}\,\text{m}^3 = 5.6 \times 10^{-6}\,\text{m}^3$

density of ABS = 1.060 $\text{g cm}^{-3} = 1.060 \times 10^{-3} \times 10^6\,\text{kg m}^{-3} = 1.060 \times 10^3\,\text{kg m}^{-3}$

Step 2: mass = density × volume

mass per case = $1.060 \times 10^3 \times 5.6 \times 10^{-6} = 5.936 \times 10^{-3}$ kg

mass of batch = $20\,000 \times 5.936 \times 10^{-3} = 118.72$ kg

Step 3: cost of batch = 118.72 kg × £0.55/kg = £65.296 = £65.30 (nearest penny)

Answer: The cost of ABS to manufacture a batch of 20 000 remote control casings is £65.30 to the nearest penny.

2 The volume of the hemisphere can be calculated as follows:

$$\text{volume of hemisphere} = \frac{2}{3}\ \text{curved surface area} \times \text{height}$$

$$= \frac{2}{3}\pi r^3 = \frac{2}{3}\ \pi \times 20^3 = 16\,755.16\ \text{mm}^3\ \text{(to 2 d.p.)}$$

$$\text{volume of a truncated cone} = \frac{1}{3}\pi h\left(Rr + R^2 + r^2\right)$$

For the truncated cone in this question, the large radius, R, is 20 mm. The small radius, r, is 10 mm. The height, h, is not given and needs to be calculated. The slant height is known. A triangle exists, as shown in Figure A.2.

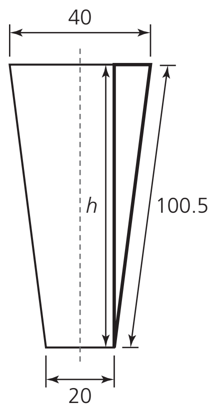


Figure A.2

From Section 4.1, Pythagoras' theorem can be used to find the height, h.

$$10^2 + h^2 = 100.5^2$$

Rearranging the formula gives:

$$h = \sqrt{100.5^2 - 10^2} = 100\ \text{mm}$$

Substituting this and the other values into the formula gives:

$$\text{volume of a truncated cone} = \frac{1}{3}\pi h\left(Rr + R^2 + r^2\right) = \frac{1}{3}\pi \times 100\left(20 \times 10 + 20^2 + 10^2\right)$$

$$= 73303.83\ \text{mm}^3\ \text{(to 2 d.p.)}$$

For the small cylinder, the volume is:

$$\text{volume of a cylinder} = \pi r^2 h = \pi \times 5^2 \times 30 = 2356.19\ \text{mm}^3\ \text{(to 2 d.p.)}$$

Total volume = 16 755.16 + 73 303.83 – 2356.19 = 87 703 mm^3 to the nearest mm^3.

87 703 mm^3 of aluminium is required to cast the counter-balance.

2720 kg m^{-3} is the same as 0.00272 g mm^{-3}.

$$\text{mass of counter-balance} = 87703\ \text{mm}^3 \times 0.00272\ \text{g mm}^{-3} = 238.55\ \text{g (to 2 d.p.)}$$

Answer: 239 g of aluminium is required to cast the counter-balance, to the nearest g.

Practice question (p. 57)

3 **a** When calculating the mass of an object, the density formula is used:

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{v}$$

Step 1: Calculate the volume of aluminium added.

The web is a cuboid, so the volume is width × height × length.

$\text{volume} = 7.00 \times 2.00 \times 2000 = 28\,000\,\text{mm}^3$

Step 2: Convert to base units of m.

$28\,000\,\text{mm}^3 = 2.8 \times 10^4\,\text{mm}^3 = 2.8 \times 10^{-5}\,\text{m}^3$

Step 3: Rearrange the density formula to make mass the subject.

mass = density × volume

Substitute the values:

$\text{mass} = 270 \times 2.8 \times 10^{-5} = 0.00756 = 7.56 \times 10^{-3}\,\text{kg} = 7.56\,\text{g}$

Answer: The mass added is only 7.56 g.

b Step 1: Convert the cost/mass units to cost/kg.

1 tonne = 1000 kg

Therefore, cost/kg = £1400/1000 kg = £1.40/kg.

Step 2: Multiply by the mass to calculate the additional cost.

Additional cost = £1.40p/kg × $7.56 \times 10^{-3}\,\text{kg}$ = 1.058p = 1p to the nearest penny.

Answer: The additional aluminium will cost only 1 pence (to the nearest penny).

4 Use of trigonometry

4.1 Pythagoras

Guided question (p. 61)

$l^2 = 12.5^2 + 22.5^2$

$= 156.25 + 506.25$

$= 662.5$

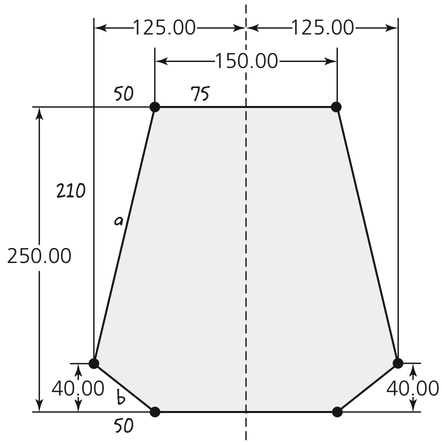
$l = \sqrt{662.5}$

$l = 25.739 = 25.7\,(1 \text{ d.p.})$

Practice question (p. 61)

2 The profile of the lampshade can be used to find the length of one strip. The left-hand and right-hand edges show the profile of a strip.

Step 1: Annotate the drawing and identify the two right-angled triangles that contain the sides of the profile that make up the length of one strip. Label the sides that need to be calculated.



Step 2: Calculate side *a* using Pythagoras' theorem.

$$a = \sqrt{50^2 + 210^2} = 215.87\text{mm}\ (2 \text{ d.p.})$$

Step 3: Calculate side *b* using Pythagoras' theorem.

$$b = \sqrt{40^2 + 50^2} = 64.03\text{mm}\ (2 \text{ d.p.})$$

Step 4: Add *a* and *b* to calculate the length of a strip of polypropylene.

Strip of polypropylene = 215.87 + 64.03 = 279.90 mm (2 d.p.) = 280 mm (nearest mm).

Answer: The length of a polypropylene strip is 280 mm (to the nearest mm).

4.2 Sine, cosine and tangent

Guided questions (p. 66)

1 $\tan a = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{12.5}{22.5} = 0.556\,(\text{to 3 s.f.})$

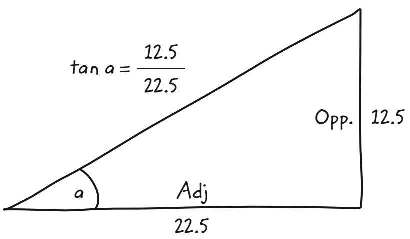


Figure A.3

$$a = \tan^{-1}0.556 = 29.05 = 29° \text{ to the nearest degree}$$

Answer: The angle for taper turning, a, is 29° to the nearest degree.

Interestingly, this ratio is very close to the following memorable ratio:

$$\tan 30 = \frac{\sqrt{3}}{3} = 0.577 \text{ (to 3 s.f.)}$$

This implies the answer is correct.

2 As shown in Figure 4.16, the rectangles will be 65 mm × 10 mm. There are six rectangles, so their area is:

$$\text{area of rectangles} = 6 \times 65\,\text{mm} \times 10\,\text{mm} = 3900\,\text{mm}^2$$

From Section 3.1 use the following formula:

$$\text{area of a regular polygon} = \frac{\text{perimeter} \times \text{apothem}}{2}$$

The hexagon can be divided into six equilateral triangles. Sketch the apothem on the diagram to help you, as shown in Figure A.4.

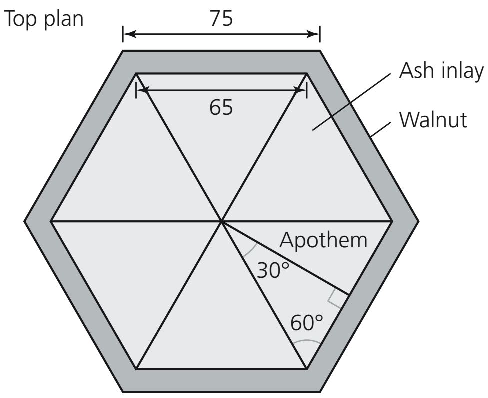


Figure A.4

Using SOHCAHTOA, identify that the apothem can be found by using the sine ratio.

$$\text{apothem} = \sin 60° \times \text{hypotenuse of right-angled triangle}$$

Remember that $\sin 60° = \frac{\sqrt{3}}{2}$

$$\text{apothem} = \frac{\sqrt{3}}{2} \times \text{ hypotenuse}$$

The triangles that make up the hexagon are all equilateral and thus each side is 65 mm. Therefore:

$$\text{area of hexagon} = \frac{\text{perimeter} \times \text{apothem}}{2} = \frac{6 \times \text{side}}{2} \times \frac{\sqrt{3} \times \text{side}}{2} = \frac{3\sqrt{3}\ \text{side}^2}{2}$$

$$= \frac{3\sqrt{3} \times 65^2}{2} = 10\,976.87\,\text{mm}^2 \text{ (to 2 d.p.)}$$

Total area = 3900 + 10 976.87 = 14 876.87 mm^2 (to 2 d.p.)

To calculate the cost of the ash veneer used, the cost per unit area of the ash veneer in £/mm^2 is required.

Area of sheet of ash veneer = 2750 mm × 200 mm = 550 000 mm^2

$$\text{Cost/area} = \frac{£11.10}{550\,000} = 0.0000202 \text{ (to 3 s.f.)}$$

Cost of veneer = total area of veneer used × cost/area

= 14 876.87 × 0.0000202 = £0.30 to the nearest pence

Answer: The cost of the ash veneer required for the jewellery box lid is £0.30 to the nearest pence.

3 Step 1:

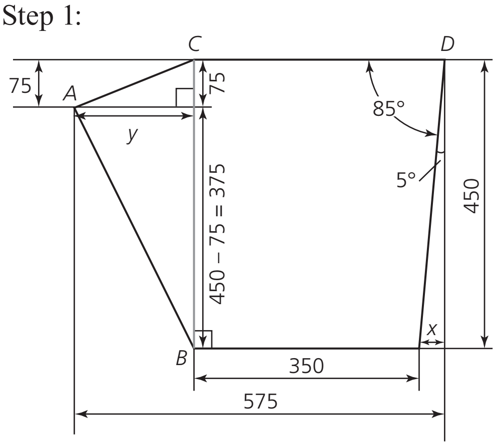


Figure A.5

Step 2: To calculate x, use the tangent ratio:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Rearranging and substituting values gives:

opposite, $x = \tan\theta \times \text{adjacent} = \tan 5° \times 450 = 39.37$ mm (2 d.p.)

Step 3: $y = 575 - 350 - x$

Substituting $x = 39.37$ gives:

$y = 575 - 350 - 39.37$

$y = 185.63$ mm (2 d.p.)

Step 4: Pythagoras' theorem can be used for this.

$a^2 + b^2 = c^2$

Substituting values and rearranging gives:

$$AB^2 = 185.63^2 + 375^2 = 175083.50 \text{ (2 d.p.)}$$

$AB = 418.43$ mm (2 d.p.)

i Answer: The length of line AB is 418 mm (to the nearest mm).

Step 5: $CD = 350$ mm + 39.37 mm = 389.37 mm (2 d.p.)

ii Answer: The length of line CD is 389 mm (to the nearest mm).

Practice questions (p. 67)

4 a First, sketch the desktop and enveloping rectangle.

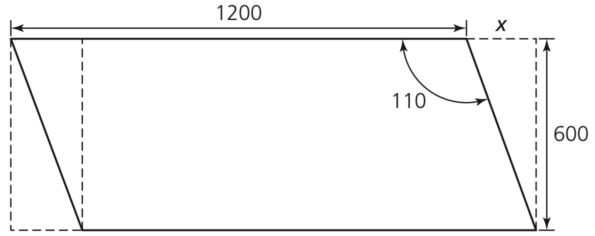


Figure A.6

The length x is needed. You are given the length of a side and an angle can be deduced from the angle of the walls. Using SOHCAHTOA to remember, this can be calculated by using the tangent ratio.

$$\tan\left(180° - 110°\right) = \frac{600}{x}$$

$$x = \frac{600}{\tan 70°} = 218.38\text{mm (to 2 d.p.)}$$

The rectangle of oak-veneered MDF therefore needs to be: 1418 mm × 600 mm.

Answer: 1418 mm long × 600 mm wide (to nearest mm).

b The cost of the area of the rectangle as a proportion of the cost of a full board needs to be calculated.

area of rectangle = 1418 mm × 600 mm = 850 800 mm^2

A standard-sized manufactured board is 2440 mm × 1220 mm = 2 976 800 mm^2.

$$\text{cost of rectangle} = \text{cost of full board} \times \frac{\text{area of rectangle}}{\text{area of full board}} = £69 \times \frac{850\,800}{2\,976\,800}$$

= £19.72 to the nearest penny.

Answer: The cost of the rectangle of oak-veneered MDF from which the parallelogram-shaped desktop will be made is £19.72 to the nearest penny.

c The length of the parallel sloping edges of the desktop is needed to complete this calculation. It can be calculated using Pythagoras' theorem because you have the lengths of two sides of the small right-angled triangle, or by using the sine or cosine ratios.

Method 1: using Pythagoras' theorem

$$A = \sqrt{B^2 + C^2}$$

$$\text{hypotenuse} = \sqrt{600^2 + 218.38^2} = 639 \text{ mm to nearest mm}$$

Method 2: using sine ratio

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{hypotenuse} = \frac{600\text{ mm}}{\sin\left(180° - 110°\right)} = 639 \text{ mm to nearest mm}$$

Method 3: using cosine ratio

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{hypotenuse} = \frac{218.38\,\text{mm}}{\cos(180° - 110°)} = 639\,\text{mm to nearest mm}$$

The total surface area that has an application of French polish is the sum of the areas of the sides and the top.

total surface area = (2 × (sloping edge × 19 mm)) + (2 × (length × 19 mm)) + (1200 × 600)

= (2 × (639 × 19)) + (2 × (1200 × 19)) + (1200 × 600)

= 789 882 mm^2

This surface area receives two coats of French polish, so the total surface area covered is 1 579 764 mm^2.

1 litre of lacquer covers 8 m^2 or 8 000 000 mm^2.

$$\text{cost of french polish} = \frac{\text{cost of 1 litre}}{\text{area covered by 1 litre}} \times \text{area to be covered}$$

$$= \frac{£32}{8\,000\,000} \times 1\,579\,764$$

= £6.32 to the nearest penny.

Answer: The cost of French polish required to provide two coats to the desktop and sides is £6.32 to the nearest penny.

5 Step 1: Redraw the top left triangle formed by the 740 mm dimension, table top and leg. Identify the dimensions required.

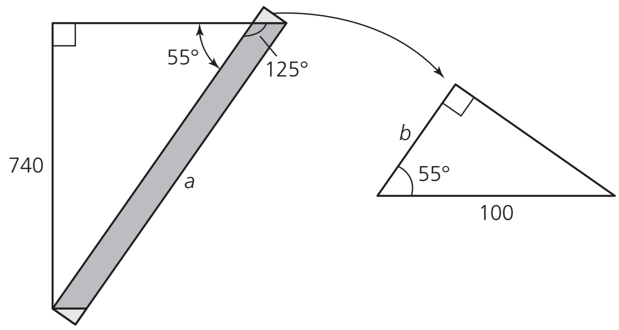


Figure A.7

In Figure A.7, the missing dimensions required make up the leg length, which is $a + b$.

Step 2: Identify the relationship between the given angle of 55°, the leg and triangle made by the missing piece of leg that forms a triangle. This provides sufficient information to calculate length b.

Step 3: Calculate length a using the sine ratio.

$$\sin\phi = \frac{\text{opposite}}{\text{hypotenuse}}$$

Substitute values:

$$\sin 55° = \frac{740}{a}$$

Rearrange to make a the subject:

$$a = \frac{740}{\sin 55°} = 903.37\,\text{mm}\ (2\text{ d.p.})$$

Step 4: Calculate the length of b using the cosine ratio.

$$\cos\phi = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute values:

$$\cos 55° = \frac{b}{100}$$

Rearrange to make b the subject:

$$b = 100\cos 55° = 57.36\,\text{mm}\ (2\text{ d.p.})$$

Step 5: Add a and b to find the length of the leg.

Leg length = 903.37 + 57.36 = 961 mm (3 s.f.)

Answer: The length of each oak leg is 960 mm.

6 Step 1: For any prism, the volume can be calculated by multiplying the area of the cross-section by the height. Both sides of the star that are given are of equal length. They form an equilateral triangle with a pentagon at the centre of the star, as shown in grey in Figure A.8.

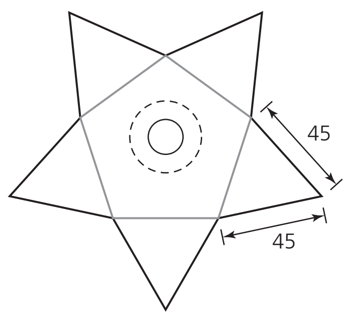


Figure A.8

The volumes of the holes were calculated in an earlier worked example and are given in the question.

volume of the star = area of star × height, 30 mm

First, calculate the area of the star:

area of the star = area of 5 × equilateral triangles + area of pentagon

Step 2: In order to calculate the area of a triangle, the height, B, of the triangle is needed (see Figure A.9).

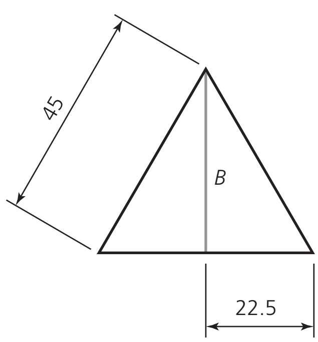


Figure A.9

Pythagoras' theorem, from Section 4.1, gives:

$$B = \sqrt{A^2 - C^2}$$

Substituting values gives (see Figure A.10):

$$B = \sqrt{45^2 - 22.5^2} = 39 \text{ mm } (2 \text{ s.f.})$$

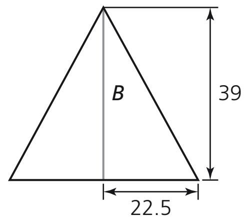


Figure A.10

Step 3: Calculate the area of the equilateral triangle.

$$\text{area of equilateral triangle} = \frac{\text{width, } w \times \text{height, } h}{2}$$

For the equilateral triangles in the star, $h = B$ and $w = 45$ mm.

Substituting values gives:

$$\text{area of equilateral triangle} = \frac{45\text{ mm} \times 39\text{ mm}}{2} = 877.5\text{ mm}^2$$

Step 4: Calculate the area of the pentagon.

$$\text{area of regular polygon} = \frac{\text{perimeter} \times \text{apothem}}{2}$$

For the pentagon, this becomes:

$$\text{area of pentagon} = \frac{5 \times 45 \times \text{apothem}}{2}$$

Step 5: The apothem of the pentagon is the height of the isosceles triangles that form the pentagon. Trigonometry from Section 4.2 is needed to calculate the length of the apothem, *A*. The pentagon comprises five isosceles triangles as shown in Figure A.11. Note: if this was a hexagon, these triangles would be equilateral triangles.

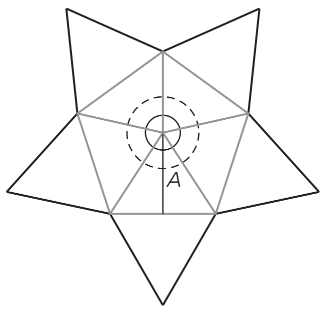


Figure A.11

Remember, from Section 3.1, that the internal angles of a pentagon are 108°. This means that each of the corner angles, *c*, of the isosceles triangle that lie against the pentagon are half of this angle, 54°.

Alternatively, angle d is easy to calculate (see Figure A.12). Angle d is half the angle at the apex of each isosceles triangle. There are five of these triangles.

$$5 \times 2d = 360°, \text{ so angle } d = \frac{360}{5 \times 2} = 36°$$

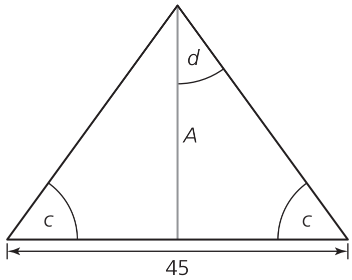


Figure A.12

Step 6: To calculate the length of the apothem, A, you use the tangent ratio:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

For the 36° angle (see Figure A.13), the opposite is 22.5 mm.

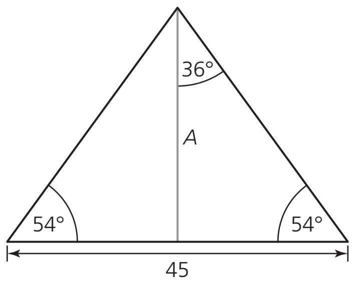


Figure A.13

Rearranging and substituting values gives A to be:

$$\text{adjacent} = \frac{\text{opposite}}{\tan\theta} = \frac{22.5}{\tan 36} = 31\,\text{mm}\ (2\text{ s.f.})$$

Step 7: Calculate the area of the pentagon.

$$\text{area of pentagon} = \frac{5 \times 45 \times \text{apothem}}{2} = \frac{5 \times 45 \times 31}{2} = 3487.5\,\text{mm}^2$$

Step 8: Calculate the area of the star.

area of the star = area of 5 × equilateral triangles + area of pentagon

Substituting values gives:

area of the star = 5 × 877.5 + 3487.5 = 7875 mm^2

Step 9: Calculate the volume of the star without holes.

volume of star = area of star × height

Substituting values gives:

volume of star = 7875 × 30 = 236 250 mm^3

Step 10: Calculate the volume of the holes to find the answer to the question.

volume of small hole $= 3141\,\text{mm}^3$

volume of large hole $= 25132\,\text{mm}^3$

Step 11: Calculate the volume of aluminium used.

volume of aluminium = volume of star-shaped prism – volume of holes

Substituting values:

$$\begin{aligned}\text{volume of aluminium} &= 236\,250\,\text{mm}^3 - 3141\,\text{mm}^3 - 25132\,\text{mm}^3 = 207\,977\,\text{mm}^3\\ &= 2.1 \times 10^5\,\text{mm}^3 (2\text{ s.f.}) = 2.1 \times 10^{-4}\,\text{m}^3 (2\text{ s.f.})\end{aligned}$$

Answer: The volume of aluminium needed to cast the base is $2.1 \times 10^{-4}\,\text{m}^3$ (2 s.f.).

4.3 Sine and cosine rules

Guided questions (p. 71)

1 Step 1: Using the diagram of the base triangle (Figure 4.29):

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute values:

$$\cos 30° = \frac{14\,\text{mm}}{C}$$

Remember, $\cos 30° = \frac{\sqrt{3}}{2}$

$$\frac{\sqrt{3}}{2} = \frac{14}{C}$$

Rearrange to make C the subject:

$$C = \frac{14 \times 2}{\sqrt{3}} = 16.17\,\text{mm}\,(2\text{ d.p})$$

Step 2: Using Pythagoras' theorem on the vertical triangle (Figure 4.28):

$$30^2 + C^2 = u^2$$

$$30^2 + 16.17^2 = u^2$$

Rearrange to make u the subject:

$$u = \sqrt{30^2 + 16.17^2} = 34.08\,\text{mm}\,(2\text{ d.p.})$$

Step 3: Using the cosine ratio on the side triangle (Figure 4.30):

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Substitute values:

$$\cos a = \frac{14\,\text{mm}}{u} = \frac{14\,\text{mm}}{34.08\,\text{mm}}$$

Rearrange to make a the subject:

$$a = \cos^{-1}\frac{14}{34.08} = 65.75° \text{ (2 d.p.)}$$

Answer: Angle a is 66° (2 s.f.).

Step 4: Calculate angle b using the sine ratio on the vertical triangle.

$$\sin b = \frac{30}{34.08} = 0.88 \text{ (2 d.p.)}$$

$$b = \sin^{-1} 0.88 = 61.68° \text{ (2 d.p.)}$$

Answer: Angle b is 62° (2 s.f.).

2 Step 1: In the example in Figure 4.32, the missing dimensions are shown as a, b and c. a is not required for the final calculation because only the perimeter of the lightning strike is cut by the laser.

Step 2: Using the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Substitute values from the triangular shape:

$$\frac{b}{\sin 20°} = \frac{100}{\sin 110°}$$

Rearrange to give:

$$b = \frac{100}{\sin 110°} \times \sin 20° = 36.40 \text{ mm (2 d.p.)}$$

Step 3: Using the sine rule again:

$$\frac{(a+c)}{\sin 50°} = \frac{100}{\sin 110°}$$

Rearrange to give:

$$a + c = \frac{100}{\sin 110°} \sin 50° = 81.52 \text{ mm (2 d.p.)}$$

Step 4: Label the angles e and f on Figure 4.33 that are needed to calculate a.

Step 5:

$$d = \sqrt{75^2 + 30^2} = 80.78 \text{ mm (2 d.p.)}$$

Step 6: Use the sine rule to calculate angle e:

$$\frac{80.78}{\sin 105°} = \frac{24.75}{\sin e}$$

Rearrange to give:

$$\text{angle } e = \sin^{-1}\frac{\sin 105° \times 24.75}{80.78} = 17.21° \text{ (2 d.p.)}$$

Step 7: $180° = 105° + e + f$

Rearrange to make angle f the subject:

$$\text{angle } f = 180° - 105° - 17.21° = 57.79° \text{ (2 d.p.)}$$

Step 8:

$$\frac{a+50}{\sin 57.79°} = \frac{80.78}{\sin 105°}$$

Rearrange to give:

$$a+50 = \frac{80.78}{\sin 105°}\sin 57.79° = 70.76\,\text{mm (2 d.p.)}$$

Step 9: Length of line a = 70.76 – 50 = 20.76 mm (2 d.p.)

Step 10: Length of line c = 81.52 – 20.76 = 60.76 mm

Step 11: Perimeter = 100 + 60.76 + 24.75 + 75 + 30 + 50 + 36.40 = 376.91 mm (2 d.p.)

= 377 mm (3 s.f.)

Step 12:

$$\text{time} = \frac{\text{perimeter}}{\text{speed}} = \frac{377}{3.5} = 107.71 = 108 \text{ seconds (3 s.f.)}$$

Answer: It will take 108 seconds (3 s.f.) to cut the lightning strike out of the 3 mm acrylic.

Practice question (p. 73)

3 area of regular polygon = $\frac{\text{perimeter} \times \text{apothem}}{2}$

Step 1: To calculate the area of the groundsheet, the apothem is required. Sketch out the decagon and add all known dimensions from your knowledge of the properties of a regular polygon (see Figure A.14).

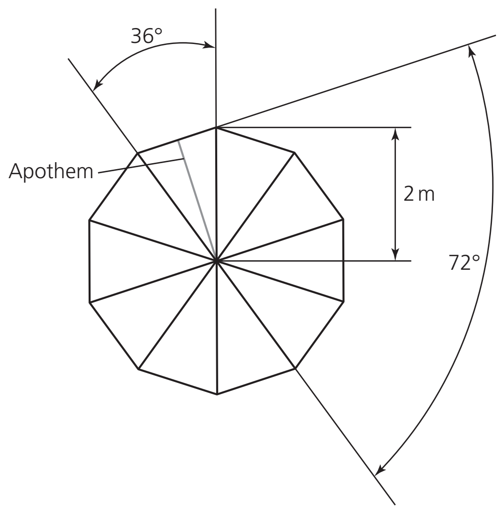


Figure A.14

For any regular polygon, the angles of the isosceles triangle apexes must add up to 360°. Therefore, one apex of a decagon is one tenth of this, i.e. 36°.

The internal angles of all triangles add up to 180°. Therefore, the remaining angles of the isosceles triangle add up to 180° – 36° = 144°. Theses angles are equal and thus are 72° each.

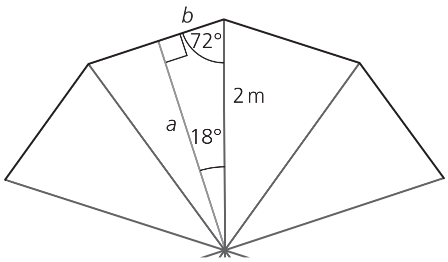


Figure A.15

Step 1: Calculate the length of one side of the decagon, which is $2 \times b = 2b$, in order to calculate the perimeter. This value will also be used to find the apothem.

$$\sin 18 = \frac{b}{2}$$

$$b = 2 \sin 18$$

$$b = 0.6180 \text{ m} (4 \text{ s.f.})$$

Step 2: Calculate the perimeter.

perimeter = $10 \times 2b = 10 \times 1.236 = 12.36$ m (4 s.f.)

Step 3: Calculate the apothem, a. Again, the cosine ratio can be used as the short side of the triangle is now known.

$$\cos 18° = \frac{a}{2}$$

$$a = 2 \times \cos 18° = 1.902 \text{ (4 s.f.)}$$

$$a = 1.902 \text{ m (4 s.f.)}$$

Check: Pythagoras' theorem can also be used as the apothem is at right angles to side b.

$$a = \sqrt{c^2 - b^2}$$

Substitute values:

$$a = \sqrt{2^2 - 0.6180^2} = 1.902 \text{ m} (4 \text{s.f.})$$

Step 4: Calculate the area of the groundsheet.

$$\text{area of regular polygon} = \frac{\text{perimeter} \times \text{apothem}}{2}$$

Substitute values:

$$\text{area of decagon} = \frac{12.36 \times 1.902}{2} = 11.75 \text{ m}^2 = 12 \text{ m}^2 (2 \text{ s.f.})$$

Answer: The area of the groundsheet is 12 m^2 (2 s.f.).

You could also solve this knowledge using basic knowledge of geometry, triangles and trigonometry. The decagon is made up from ten identical isosceles triangles, and the area of one of the triangles is first calculated (one can deduce that the long side of the triangle is 200 cm, and the apex angle is 36 degrees. Trigonometry is then used to calculate the vertical height of the triangle, and the formula $0.5 \times \text{base} \times \text{height}$ is used to calculate area). This area is then multiplied by ten to find the total area of the groundsheet.

4.4 Direction of movement

(OCR Design Engineering only, *online*)

Guided question (p. 22)

1 Step 1: Mark the unknown height h of the jet on your diagram.

Step 2: The water leaves the jet horizontally at $8\,\text{m}\,\text{s}^{-1}$. Therefore:

initial vertical component of velocity, $u_{\text{vertical}} = 0$

initial horizontal component of velocity, $u_{\text{horizontal}} = 8\,\text{m}\,\text{s}^{-1}$

Step 3: In projectile motion, the horizontal component of velocity stays constant, at $8\,\text{m}\,\text{s}^{-1}$. Therefore, we can use the equation:

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

Rearrange to find time:

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

Substitute in the values:

$$\text{time} = \frac{4}{8} = 0.5\,\text{s}$$

Step 4: In the 0.5 s it takes the water to travel 4 m horizontally, we can calculate the distance s that the water will fall vertically. This will be the height at which the jet needs to be mounted above the ground.

In this step, we are dealing with vertical motion only. Write down the parameters that we already know:

$u_{\text{vertical}} = 0$ (refer back to step 2)

$t = 0.5\,\text{s}$

$a = g = 9.81\,\text{m}\,\text{s}^{-2}$

We need to find s.

Since this problem deals only with vertical motion in a downwards direction, we can take downwards as being positive. This means that g will have a positive value, as it also acts downwards. Use the equation of motion:

$$s = ut + \frac{1}{2}at^2$$

Substitute in the values:

$$s = 0 + (\frac{1}{2} \times 9.81 \times 0.5^2)$$

$s = 1.23\,\text{m}$ (to 2 d.p.)

Practice questions (p.23)

2 The ball is projected horizontally at a velocity which we need to find. However, we know that the initial vertical component of the ball's velocity must be zero.

Step 1: Find the time taken for the ball to fall through a vertical distance of 0.5 m.

Write down the parameters that we know. In this step, we are dealing with vertical motion only and taking the downwards direction as positive:

$u_{vertical} = 0$

$s = 0.5\,\text{m}$

$a = g = 9.81\,\text{m}\,\text{s}^{-2}$

We need to find t. Use the equation:

$$s = ut + \frac{1}{2}at^2$$

Before we rearrange the equation to find t, it is worth realising that the first term ($u \times t$) is zero because the initial vertical velocity is zero. Therefore, the equation rearranges to:

$$s = \frac{1}{2}at^2$$

$$t^2 = \frac{2s}{a}$$

$$t = \sqrt{\frac{2s}{a}}$$

Substitute in the values:

$$t = \sqrt{\frac{2 \times 0.5}{9.81}}$$

$t = 0.319\,\text{s}$ (to 3 d.p.)

Now let's deal with the horizontal motion. The question tells us that the ball must travel 2.0 m horizontally and we have already calculated that it must travel this distance in 0.319 s. Therefore, we can easily calculate the horizontal velocity using:

$$\text{horizontal velocity} = \frac{\text{horizontal distance}}{\text{time}}$$

$$\text{horizontal velocity} = \frac{2.0}{0.319}$$

horizontal velocity $= 6.27\,\text{m}\,\text{s}^{-1}$ (to 2 d.p.)

3 Resolve the bicycle's initial velocity into vertical and horizontal components:

$u_{vertical} = 6 \times \sin(10°) = 1.04\,\text{m}\,\text{s}^{-1}$ (to 2 d.p.)

$u_{horizontal} = 6 \times \cos(10°) = 5.91\,\text{m}\,\text{s}^{-1}$ (to 2 d.p.)

We need to find the time for which the rider stays airborne before landing back on the ground. The easiest way to do this is to calculate the time taken for the rider to reach maximum height and then double this time because it will take the same amount of time for the rider to reach the ground again.

Dealing with vertical motion only, we will calculate the time taken to reach maximum height. Write down the parameters we know:

$u_{vertical} = 1.04\,\text{m}\,\text{s}^{-1}$ (this is upwards, therefore positive)

$v_{vertical} = 0$ (since the rider's vertical motion will have stopped when they reach maximum height)

$a = g = -9.81\,\text{m}\,\text{s}^{-2}$ (this acts downwards, therefore negative)

We need to find t. Use the equation:

$$v = u + at$$

Rearrange to find t:

$$v - u = at$$

$$t = \frac{v - u}{a}$$

$$t = \frac{0 - 1.04}{-9.81}$$

$t = 0.106\,\text{s}$ (to 3 d.p.)

Therefore, the time the rider stays airborne:

$$T = 2 \times 0.106 = 0.212\,\text{s}$$

We can now use this time to calculate the horizontal distance travelled. Remember that the horizontal velocity stays constant:

horizontal distance = horizontal velocity × time

horizontal distance = 5.91 × 0.212 = 1.25 m

4.5 Resolving force vectors

(OCR Design Engineering only, *online*)

Guided question (p. 24)

1 Step 1: Draw a force diagram.

Step 2:

$F_{vertical} = F\cos(50°)$ (we need to find the force F)

$F_{horizontal} = F\sin(50°)$ (we can ignore this horizontal component as it acts along the lever, through the fulcrum)

Step 3: Calculate the mechanical advantage of the lever:

$$\text{mechanical advantage} = \frac{\text{input arm length}}{\text{output arm length}}$$

$$\text{mechanical advantage} = \frac{200}{400} = 0.5$$

The mechanical advantage is also equal to:

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

We know that the load is 5 N.

Rearrange the equation to find the effort:

$$\text{effort} = \frac{\text{load}}{\text{mechanical advantage}}$$

$$\text{effort} = \frac{5}{0.5} = 10\,\text{N}$$

The effort force is the vertical component of force F. Therefore, using the result from step 2:

$$F\cos(50°) = 10$$

$$F = \frac{10}{\cos(50°)} = 15.6\,\text{N (to 1 d.p.)}$$

Practice questions (p. 25)

2 Step 1: Use the dimensions given in the question to calculate the angle of strut S:

$$\tan\theta = \frac{100}{300}$$

$\theta = 18.4°$ (to 1 d.p.) (this is the angle between strut S and the horizontal)

For the top end of strut S to be in equilibrium, the upward forces at this point must equal the downward forces. (Also, at the same point, the forces to the left must equal forces to the right, but we don't need to concern ourselves with horizontal forces in this problem.)

downward force at top of strut S = weight applied (call this W)

upward force at this point = vertical component of force in strut S

When the force in strut S is 50 N:

$$W = 50 \times \sin(18.4°)$$

$$W = 15.8\,\text{N (to 1 d.p.)}$$

3 This question just requires us to calculate the component of the cart's weight that is acting parallel to the slope, as this is the force that is required to keep the cart moving at a steady speed.

If you draw a diagram of the forces acting on the cart, you should be able to see that the component of the 500 N weight that acts parallel to the slope is:

$$F = 500 \times \sin(8°)$$

$$F = 69.6\,\text{N (to 1 d.p.)}$$

5 Use and analysis of data, charts and graphs

5.1 Presenting data

Practice question (p. 75)

1 **a** five

b 99 900

c 150 thousand

d two

e 23

5.2 Statistics

Guided question (p. 77)

1 **a** The mode for type is 'Clamshell', which occurs ten times in the sample.

b Both 13.3″ and 12.0″ occur the most, with four occurrences in the sample, so there is no mode.

c Intel Skylake Core U appears the most with six occurrences, so this is the mode for CPU.

Practice question (p. 78)

2 **a** For this sample:

$$\text{mean average} = \frac{\text{sum of values in sample}}{\text{number of values in sample}}$$

$$\text{mean average} = \frac{0.69+0.73+0.77+0.78+0.79+0.84+0.84+0.87+0.91+0.91+0.91+0.91+0.92+0.95+0.98+0.98+0.98+0.99+0.99}{19}$$

$$= 0.881$$

Answer: The average (mean) weight of laptop is 0.88 kg (2 s.f.).

b For a sample with an odd number of values, the median is the middle value. The middle value of a sample of 19 values is the tenth value. The tenth value is 0.91 kg. Therefore, the answer to the question is the models of laptops that have this weight.

Answer: Asus Zenbook UX390UA, Apple MacBook 12, Asus Chromebook Flip and Asus Chromebook C201 best represent the median weight of the sample.

c This is the value that occurs the most often. 0.91 kg is the value that appears the most, with four occurrences.

Answer: 0.91 kg is the mode weight of the sample.

5.3 Group data, estimates, modal class and histograms

Guided question (p. 83)

1 Step 1: The lower class boundaries are 10, 20, 30, 40, 50, 60, 70 and 80.

Step 2: In this case, all the class widths are the same, i.e. 30 – 20 = 10 is the same as 20 – 10 = 10. The class widths are 10.

Step 3:

$$\text{area} = \text{frequency} = \text{frequency density} \times \text{class width}$$

Therefore:

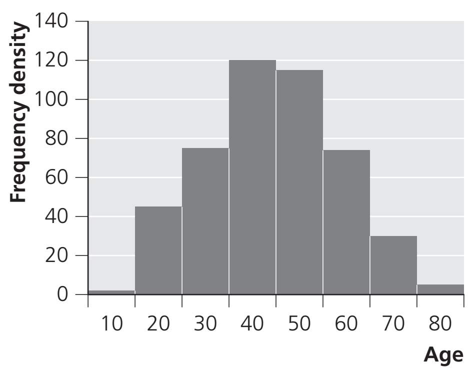
$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

It is a good idea to create a table when you calculate these densities, similar to Table A.1.

Table A.1 Data for histogram

Age	Lower class limit	Frequency	Frequency density
80–89	80	50	5
70–79	70	300	30
60–69	60	740	74
50–59	50	1150	115
40–49	40	1200	120
30–39	30	750	75
20–29	20	430	43
10–19	10	20	2

Step 4:



5.4 Presenting market and user data

Guided questions (p. 87)

1 Step 1: The data provided is not suited to a line chart, which is best for presenting data of an analogue nature, such as time. Bar charts or pie graphs would be most suitable. Pie graphs are best to show extreme differences in values.

It would be valuable to present the number of laptops of similar weights measured. This would help identify the most popular weights and the mode.

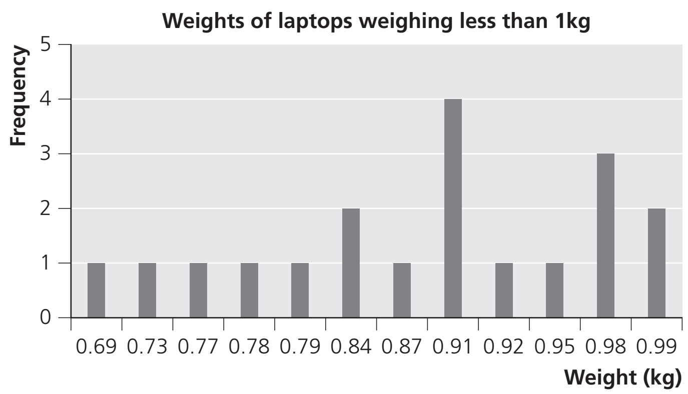
Step 2:

Table A.2 Weight of laptops

Weight (kg)	Frequency
0.69	1
0.73	1
0.77	1
0.78	1
0.79	1
0.84	2
0.87	1
0.91	4
0.92	1
0.95	1
0.98	3
0.99	2

Step 3: A pie chart would be confusing and require time to interpret. The bar chart below instantly conveys the message that 0.91 kg is the most popular weight of the sample.

Answer: Bar chart



2 Step 1:

Table A.3 Weight of laptops by screen size

Screen	Weight (kg)
10.1	0.91
11.6	0.87
11.6	0.91
12	0.77
12	0.79
12	0.91
12	0.95
12.1	0.69
12.3	0.78
12.5	0.91
12.5	0.99
12.5	0.99
13.3	0.84
13.3	0.84
13.3	0.92
13.3	0.98
13.5	0.73
14	0.98
15.6	0.98

Step 2: An appropriate chart type is a scatter graph. This will enable the weight to be plotted against screen size and any trend identified through the grouping of the scatter graph marks.

The graph in Figure A.16 has both axis range limits set to best present the data, i.e. the *x*-axis range is from 9″ up to 16″ and the *y*-axis range is 0.6 kg to 1.05 kg. Your spreadsheet application will allow you to make these changes using graph axis format tools.

Answer: Scatter graph.

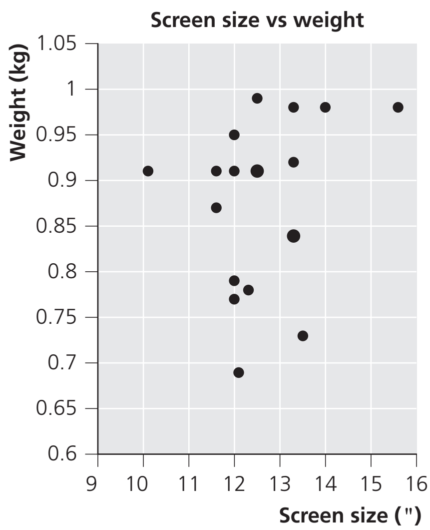


Figure A.16

5.6 Interpret statistical analyses to determine user needs

Guided question (p. 91)

1 **a** Number of students in class = (1 + 4 + 8 + 3 + 7 + 4 + 3 + 2) = 32.

b It is clear from the bar chart that the 135–139 cm group is the most common.

c Height range = 164 – 125 = 39 cm.

d The mean is calculated in the normal way by adding the mid-category heights of all the students and then dividing by the total number of students:

sum of heights = $(1 \times 127) + (4 \times 132) + (8 \times 137) + (3 \times 142) + (7 \times 147) + (4 \times 152) + (3 \times 157) + (2 \times 162) = 4609$ cm

mean height = $\frac{4609}{32} = 144$ cm (rounded to nearest cm)

Practice questions (p. 91)

2 **a** From Figure 5.19 it can be seen that console (at 30%) is the second most used gaming device, compared with phone (at 40%).

b 120 people were surveyed. From the figure, 10% of people use a laptop. Calculate 10% of 120:

$120 \times 0.1 = 12$ people

c The ratio of people is the same as the ratio of the percentages in the chart.

console : PC = 30 : 15 = 2 : 1

d percentage of phone users = 40%

percentage of console users = 30%

Find the mean of these two values:

mean value = $\frac{40 + 30}{2} = 35\%$

Therefore, 5% of the total number of users need to move from phone to console to make them both equal. Calculate 5% of 120:

$120 \times 0.05 = 6$ people

3 **a** To find the total number of people it is necessary to add the numbers in each of the age groups:

total number = 8 + 12 + 24 + 43 + 41 + 27 + 23 + 18 + 3 + 1
= 200 people

b Total number in the 30–50 age group = 43 + 41 = 84 people.

Calculate the percentage of the total:

$\frac{84}{200} \times 100 = 42\%$

c There is insufficient data to draw this conclusion. An alternative conclusion could be that there were simply more people aged 30–50 in the airport on that day.

d As the age data is grouped, we can only estimate the mean age of the voters by assuming that each voter's age sits in the centre of each group's range.

Age range	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Centre age	5	15	25	35	45	55	65	75	85	95
Number of voters	8	12	24	43	41	27	23	18	3	1

The mean is calculated by adding the ages of all the voters and then dividing by the total number of voters:

$$\text{sum of ages} = (8 \times 5) + (12 \times 15) + (24 \times 25) + (43 \times 35) + (41 \times 45) + (27 \times 55) + (23 \times 65) + (18 \times 75) + (3 \times 85) + (1 \times 95)$$

$$= 8850$$

$$\text{mean age} = \frac{8850}{200} = 44.25$$

4 **a** Busiest hour is 19–20; quietest hour is 9–10.

b Busiest : quietest = 48 : 4 = 12 : 1.

c Add up the users in the groups 7–8 and 8–9:

number of users = 25 + 35 = 60 users

d Mean users per hour is found by adding together the numbers of users in each hourly slot and dividing by the number of slots:

$$\text{sum of users in each hourly slot} = 25 + 35 + 4 + 10 + 13 + 30 + 25 + 17 + 13 + 10 + 16 + 30 + 48 + 14$$

$$= 290 \text{ users}$$

number of hourly slots = 14

$$\text{mean number per hour} = \frac{290}{14} = 20.7 \text{ per hour (to 1 d.p.)}$$

e Slots with fewer than 20.7 users per hour are: 9–10, 10–11, 11–12, 14–15, 15–16, 16–17, 17–18 and 20–21.

5.7 Graphs of motion

(OCR Design Engineering only, *online*)

Guided questions (p. 28)

1 **a** Graph B as it has a constant gradient.

b Graph D as the change in gradient from positive to negative indicates a reversal in direction of motion.

c The buggy will be accelerating and graph A indicates this.

d We would expect the velocity (and therefore the gradient) to steadily decrease to zero and then steadily increase in the opposite direction. Graph C shows this.

2 **a** At 4 s, the gradient is constant and equal to:

$$\frac{\text{change in displacement}}{\text{change in time}} = \frac{(20 - 5)}{(6 - 2)} = 3.75\ \text{m s}^{-1}$$

b average velocity = $\frac{\text{total displacement}}{\text{time taken}}$

average velocity = $\frac{25}{9}$ = 2.78 m s^{-1} (to 2 d.p.)

Practice question (p. 30)

3 **a** acceleration = $\frac{\text{change in velocity}}{\text{time taken}}$

change in velocity = acceleration × time = 0.6 × 4 = 2.4 m s^{-1}

Therefore, the graph will be a straight line, starting at the origin and reaching 2.4 ms^{-1} after 4 s. The graph will then continue horizontally (remaining at 2.4 m s^{-1}) until time reaches (4 + 5) = 9 s.

b distance travelled = area under graph

distance travelled = area of triangular section + area of rectangular section

distance travelled = ($\frac{1}{2}$ × 4 × 2.4) + (5 × 2.4)

distance travelled = 16.8 m

5.8 Engineering graphs

(OCR Design Engineering only, *online*)

Guided question (p. 32)

1 **a** Young's modulus = $\frac{\Delta\text{stress}}{\Delta\text{strain}}$

Young's modulus = $\frac{290 \times 10^{6}}{0.0014}$

Young's modulus = 2.07 × 10^{11} Pa = 207 GPa

b Maximum stress = 290 MPa.

Practice questions (p. 32)

2 The significant difference is that in the linear region, the material with the high Young's modulus will have the steeper gradient.

3 **a** Reading from the graph:

R = 13 kΩ (allow 12 kΩ to 14 kΩ).

b Reading from the graph, temperature at which R = 6 kΩ:

T = 36°C (allow 34°C to 38°C).

5.9 Waveforms

(OCR Design Engineering only, *online*)

Practice questions (p. 34)

1 A diagram of a sine wave with an amplitude of 2.5 V and a period of 20 ms:

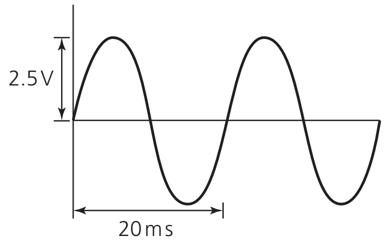


Figure A.17

2 **a** frequency $= \dfrac{\text{number of cycles}}{\text{time taken}}$

frequency $= \dfrac{30}{12} = 2.5\,\text{Hz}$

b period $= \dfrac{1}{\text{frequency}} = \dfrac{1}{2.5} = 0.4\,\text{s}$

Therefore, the square wave should have a period of 0.4 s and an amplitude of 5 V.

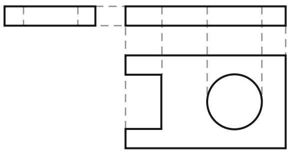
c 90° out of phase is equal to $\dfrac{90}{360} = 0.25$ of a cycle, which is 0.1 s for the waveform in this question. The waveform for the second LED will be identical to the first LED, but the second LED will rise from 0 V to 5 V a time of 0.1 s after the first LED.

6 Coordinates and geometry

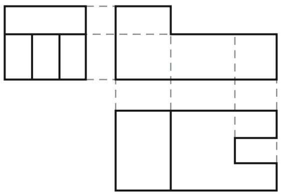
6.2 Present accurate two-dimensional and three-dimensional drawings

Guided question (p. 99)

1 **(a)**



(b)



(c)

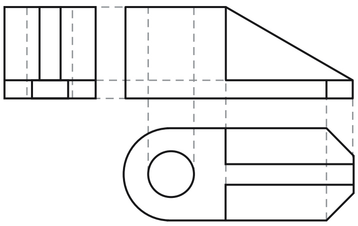


Figure A.18

Practice question (p. 101)

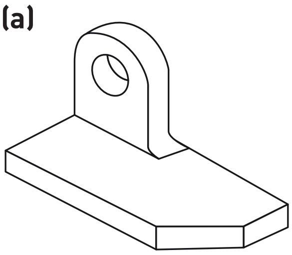
2 A copy of the shapes in Figure 6.14.

Guided question (p. 101)

3 A choice of a simple three-dimensional object.

Practice question (p. 102)

4 **(a)**



(b)

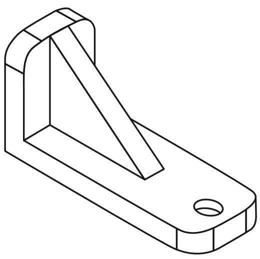


Figure A.19

7 Anthropometrics and probability

7.1 Use data related to human scale and proportion

Guided questions (p. 106)

1 $20 \times 0.05 = 1$ million people.

2 **a** From the anthropometric data, the average (50th percentile) man's thumb length is 51.0 mm. Therefore:

distance $A = 0.8 \times 51 = 40.8$ mm

b 95th percentile women's thumb width is 22.0 mm. Therefore:
minimum diameter of button B $= 0.5 \times 22 = 11$ mm

Practice questions (p. 107)

3 From the anthropometric data, the 95th percentile height is 1860 mm. 95% of the population will be shorter than this. Since the man's height is greater than the 95th percentile, then more than 95% of the population will be shorter than him.

4 **a** Adjustability over 90% of users corresponds to the range from the 5th percentile to the 95th percentile. Using the women's hip height from the anthropometric data:

seat height range = 750 mm to 890 mm

b Even at the highest setting of 890 mm, this height is less than the 50th percentile men's hip height of 935 mm. Therefore, more than half the male users have a hip height higher than the highest seat setting and are, therefore, excluded.

7.2 Understand dimensional variations in mass-produced components

Guided questions (p. 109)

1 Step 1: $\frac{330}{100} \times 5 = 16.5\,\Omega$

Step 2: Upper limit $= 330 + 16.5 = 346.5\,\Omega$

Lower limit $= 330 - 16.5 = 313.5\,\Omega$

The actual resistance will be somewhere between $313.5\,\Omega$ and $346.5\,\Omega$.

2 2.5% of 8.00 mm $= \frac{8}{100} \times 2.5 = 0.2$ mm
Catalogue shaft is 8.00 ± 0.2 mm.

The tolerance on the catalogue shaft is greater than the specified tolerance in the working drawing, so there is a risk that it will be either too small or too large. The shaft in the catalogue will not be a suitable choice in this application.

Practice questions (p. 109)

3 Calculate the percentage:

$$\frac{5}{400} \times 100 = 1.25\%$$

Tolerance = ±1.25%

4 It is necessary to find 2% of 80 mm:

$$\frac{80}{100} \times 2 = 1.6\,\text{mm}$$

The minimum length of the steel will be:

80 − 1.6 = 78.4 mm

5 Calculate 4% of 13 mm:

$$\frac{13}{100} \times 4 = 0.52\,\text{mm}$$

Tolerance = ±0.52 mm.

6 Find the minimum permissible diameter by finding 5% of 18.0 mm and then subtracting this value from 18.0 mm.

$$\frac{18}{100} \times 5 = 0.9\,\text{mm}$$

18 – 0.9 = 17.1 mm (this is the minimum permissible diameter).

17.2 mm is greater than this minimum value, so YES, the part is within tolerance.

7 Find 2% of 10.00 g:

$$\frac{10}{100} \times 2 = 0.2\,\text{g}$$

Therefore, the range of uncertainty is from 10 – 0.2 = 9.8 g to 10 + 0.2 = 10.2 g

8 Work out the minimum width for five patches. Each patch needs a 10 mm gap at the left-hand edge of the patch, plus a 10 mm gap is needed at the right-hand edge of the denim:

minimum width = (5 × (110 + 10)) + 10 = 610 mm

Work out the minimum height for eight patches. Each patch needs a 10 mm gap at the top edge of the patch, plus a 10 mm gap is needed at the bottom edge of the denim:

minimum height = (8 × (80 + 10)) + 10 = 730 mm

7.3 Probability of defects in batches and reliability

Guidance question (p. 112)

1 Step 1: $P(\text{green}) = \frac{10}{18}$

$P(\text{red}) = \frac{5}{18}$

Step 2: The probability of picking either colour is:

$P(\text{green or red}) = P(\text{green}) + P(\text{red})$

$= \frac{10}{18} + \frac{5}{18} = \frac{15}{18}$

$= \frac{5}{6}$

Practice questions (p. 114)

2 Probability of hitting bonus section:

$P(\text{bonus}) = \frac{\text{area of bonus section}}{\text{area of whole target}}$

$P(\text{bonus}) = \frac{20}{1000} = 0.02$

3 **a** There is a total of (12 + 6 + 4) = 22 LEDs.

Probability of picking a single green LED:

$P(\text{green}) = \frac{\text{number of green LEDs}}{\text{total number of LEDs}}$

$P(\text{green}) = \frac{6}{22} = 0.27$ (to 2 d.p.)

b $P(\text{green}) = 0.27$

$P(\text{red}) = \frac{4}{22} = 0.18$ (to 2 d.p.)

It is not possible to pick a single green and a single red at the same time, so the outcomes are mutually exclusive. Therefore, the probability of picking either colour is:

$P(\text{green } \textit{or} \text{ red}) = P(\text{green}) + P(\text{red})$

$P(\text{green } \textit{or} \text{ red}) = 0.27 + 0.18 = 0.45$

c Calculate the probability of picking the first blue LED:

$P(\text{blue 1}) = \frac{12}{22} = 0.55$ (to 2 d.p.)

Picking and removing the first blue LED affects the probability of picking the second blue LED, as the numbers of LEDs will have changed. Once the first blue LED has been removed, the probability of picking a second blue LED is:

$P(\text{blue 2 given blue 1}) = \frac{11}{21} = 0.52$ (to 2 d.p.)

Therefore, the probability of picking both LEDs is:

$P(\text{blue 1 } \textit{and} \text{ blue 2}) = P(\text{blue 1}) \times P(\text{blue 2 given blue 1})$

$P(\text{blue 1 } \textit{and} \text{ blue 2}) = 0.55 \times 0.52 = 0.286$

4 **a** The probability of a number 3 on either display is:

$P(3) = \frac{1}{5} = 0.2$

Generating a 3 on both displays at the same time is an independent outcome, as the generation of the first number 3 does not affect the probability of the second number 3. Therefore:

$P(3 \textit{ and } 3) = P(3) \times P(3)$

$P(3 \textit{ and } 3) = 0.2 \times 0.2 = 0.04$

b There are five outcomes in which the displays generate the same number (1, 1 or 2, 2 or 3, 3 or 4, 4 or 5, 5) and the probability of each outcome is the same. We have already calculated $P(3 \textit{ and } 3) = 0.04$, so the probability of any one of the five outcomes occurring is:

$P(\text{same number on each display}) = 5 \times 0.04 = 0.2$

5 **a** $P(\text{bike rider}) = \frac{\text{number of bike riders}}{\text{total number of children}}$.

$P(\text{bike rider}) = \frac{22}{(18 + 12)} = 0.73$ (to 2 d.p.)

b This is an independent outcome because the selection of a bike rider does not then influence whether the child is a girl or a boy. This is true because we have to assume that *P*(bike rider) is the same for girls and for boys. First, calculate the probability of selecting a girl:

$P(\text{girl}) = \frac{18}{30} = 0.6$

Therefore:

$P(\text{bike rider } \textit{and} \text{ girl}) = P(\text{bike rider}) \times P(\text{girl})$

$P(\text{bike rider } \textit{and} \text{ girl}) = \frac{22}{30} \times \frac{18}{30} = 0.44$

6 The outcome is an independent event because the selection of the battery size does not influence the choice of normal life or extended life.

$P(\text{AA}) = \frac{1}{5} = 0.2$

$P(\text{extended life}) = \frac{5}{10} = 0.5$

$P(\text{AA } \textit{and} \text{ extended life}) = P(\text{AA}) \times P(\text{extended life})$

$P(\text{AA } \textit{and} \text{ extended life}) = 0.2 \times 0.5 = 0.1$

7 $P(\text{component 1 } \textit{or} \text{ component 2}) = P(\text{component 1}) + P(\text{component 2})$

$P(\text{component 1 } \textit{or} \text{ component 2}) = 0.1 + 0.1 = 0.2$

8 This is a dependent outcome because as each faulty lamp is picked, the probability of picking another faulty one decreases. For the sake of simplicity, let's refer to the three faulty lamps as F1, F2 and F3. The probability of picking three faulty lamps is:

$P(\text{F1 } \textit{and} \text{ F2 } \textit{and} \text{ F3}) = P(\text{F1}) \times P(\text{F2 given F1}) \times P(\text{F3 given (F2 } \textit{and} \text{ F1)})$

Work out these three probabilities:

$P(\text{F1}) = \frac{25}{150} = 0.167$ (to 3 d.p.)

After selecting the first faulty lamp, the probability of selecting the second faulty lamp is:

$$P(\text{F2 given F1}) = \frac{24}{149} = 0.161 \text{ (to 3 d.p.)}$$

Then, the probability of picking the third faulty lamp is:

$$P(\text{F3 given (F2 } \textit{and} \text{ F1))} = \frac{23}{148} = 0.155 \text{ (to 3 d.p.)}$$

Therefore, the probability of picking all three faulty lamps is:

$$P(\text{F1 } \textit{and} \text{ F2 } \textit{and} \text{ F3}) = 0.167 \times 0.161 \times 0.155 = 4.17 \times 10^{-3}$$